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CALCULATION OF ROTATIONAL DERIVATIVES FOR "LOCAL" INTERACTION OF A FLOW WITH THE SURFACE OF A BODY†

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The rotational derivatives of the force and moment characteristics are calculated for solids of revolution that move at an angle of attack with small angular velocity. Formulas for rotational derivatives of the second order are derived and analysed for the general class of "local" interaction models of the flow with the surface of the body.

THE DEVELOPMENT of analytical methods of calculation for rotational derivatives in the non-translational motion of bodies in free-molecular flow is considered in [1–3]; corresponding methods for the intermediate rarefied gas flow region are developed in [2, 4, 5]. The approach proposed in [6] is intended for a fairly general class of "local" models describing the interaction of the flow with a rotating body; the implementation of this approach has led to working formulas for first rotational derivatives [6, 7]. In this paper, the proposed approach is further developed for second rotational derivatives.

In the attached coordinate system x_1, x_2, x_3 shown in Fig. 1, the expression for the radius vector of a point on the surface of the body can be represented in the form

$$\mathbf{r} = \Phi(\rho)\mathbf{x}_1^0 + \rho\cos\theta\mathbf{x}_2^0 + \rho\sin\theta\mathbf{x}_3^0$$

where x_1^0, x_2^0, x_3^0 are the unit vectors of the coordinate axes; the function $\Phi(\rho)$ defines the generator of the solid of revolution with a plane maximum middle section of radius R, and

$$\Phi(0) = 0,\ \Phi'(0) > 0,\ \Phi''(\rho) > 0,\ 0 \leqslant \rho \leqslant R,\ \Phi'(R) < \infty$$

The axes are oriented so that the translational velocity vector $\mathbf{v}_\infty$ is in the x_1, x_2 plane making an angle $\pi - \alpha$ with the x_1 axis

$$\mathbf{v}_\infty = -\mathbf{v}_\infty \cos\alpha x_1^0 + \mathbf{v}_\infty \sin\alpha x_2^0$$

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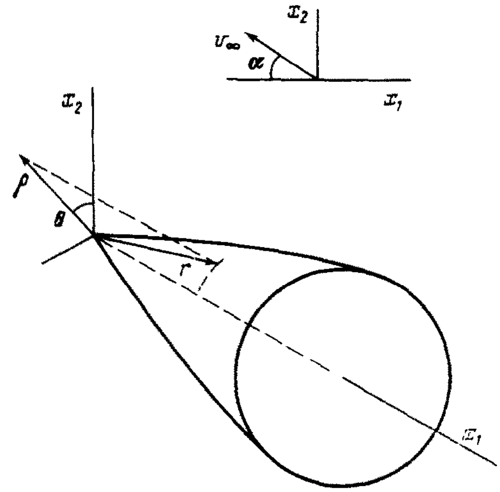

FIG. 1.

If the body rotates with angular velocity $\mathbf{v}$, the expression for the velocity $\mathbf{v}$ of a point on the surface of the body is written in the form

$$\mathbf{v} = \mathbf{v}_\infty + \boldsymbol{\omega} \times \mathbf{r}, \qquad \boldsymbol{\omega} = \sum_1^3 \omega_i \mathbf{x}_i^0$$

For the class of local interaction models [6], the expressions for the projections of the force acting on an elementary surface area along the outer normal $\mathbf{n}^0$ and the tangent $\boldsymbol{\tau}^0$ can, respectively, be represented in the form

$$dF_n = q_* \Omega_n(t) ds, \; dF_\tau = q_* (\mathbf{v} \cdot \boldsymbol{\tau}^0 / v_*) \Omega_\tau(t) ds, \; q_* = \rho_\infty v_*^2/2 \tag{1}$$

where v_* is a characteristic velocity, ρ_∞ is the unperturbed flow density and Ω_n and Ω_τ are functions that model the interaction of the flow with the body.

Changing to dimensionless parameters

$$ds/R^2 \to ds, \; \mathbf{v}/v_* \to \mathbf{v}, \; \mathbf{v}_\infty/v_* \to \mathbf{v}_\infty, \; \boldsymbol{\omega} R/v_* \to \boldsymbol{\omega}$$

and dividing all linear dimensions by R, we write expressions for the "local" force and moment coefficients relative to the origin in the form

$$d\mathbf{c}_F = (dF_n \mathbf{n}^0 + dF_\tau \boldsymbol{\tau}^0)/(q_* R^2) = [\Omega_1(t) \mathbf{n}^0 + \Omega_2(t) \mathbf{v}] ds, \; d\mathbf{c}_m = r \times d\mathbf{c}_F / R$$
$$\mathbf{n}^0 = (-\mathbf{x}_1^0 + \Phi' \cos\theta \mathbf{x}_2^0 + \Phi' \sin\theta \mathbf{x}_3^0)/\mu_1, \; t = T/\mu_1$$
$$\Omega_1(t) = \Omega_n(t) - t\Omega_\tau(t), \; \Omega_2(t) = \Omega_\tau(t)$$
$$T(\boldsymbol{\omega}, \rho, \theta) = -\mu_2 \sin\theta \omega_2 + (\mu_3 + \mu_2 \omega_3) \cos\theta + v_\infty \cos\alpha$$
$$\mu_1 = \sqrt{\Phi'^2 + 1}, \; \mu_2 = \Phi\Phi' + \rho, \; \mu_3 = v_\infty \sin\alpha \Phi'$$

After reduction, we rewrite the expressions for the total force and moment coefficients in the form

$$c_{x_i}(\boldsymbol{\omega}) = \mathbf{x}_i^0 \iint_S d\mathbf{c}_F = \iint_\sigma A_i(\boldsymbol{\omega}, \rho, \theta) \, d\rho \, d\theta \tag{2}$$

$$m_{x_i}(\boldsymbol{\omega}) = \mathbf{x}_i^0 \iint_S d\mathbf{c}_m = \iint_\sigma B_i(\boldsymbol{\omega}, \rho, \theta) \, d\rho \, d\theta \tag{3}$$

$$B_i = \rho(A_3 \cos\theta - A_2 \sin\theta)\delta_i^1 + (\rho \sin\theta A_1 - \Phi A_3)\delta_i^2 + (\Phi A_2 - \rho \cos\theta A_1)\delta_i^3 \tag{4}$$

$$\begin{aligned} A_1/\rho &= -\Omega_1(t) + \mu_1\Omega_2(t)(-v_\infty \cos\alpha + \rho\sin\theta\omega_2 + \rho\cos\theta\omega_3) \\ A_2/\rho &= \Phi'\cos\theta\Omega_1(t) + \mu_1\Omega_2(t)(\Phi\omega_3 - \rho\sin\theta\omega_1 + v_\infty\sin\alpha) \\ A_3/\rho &= \Phi'\sin\theta\Omega_1(t) + \mu_1\Omega_2(t)(\rho\cos\theta\omega_1 - \Phi\omega_2) \end{aligned} \tag{5}$$

The "illuminated" part of the body surface S is characterized by the condition $t \geqslant 0$; σ is the projection of S on the x_2, x_3 plane and δ_1^{ν} is the Kronecker delta.

Depending on the angle of attack and the shape of the body, we can have three different "illuminated" regions for small ω. When $\mathrm{ctg}\,\alpha > \Phi'(1)$, the lateral surface is fully "illuminated". When $\Phi'(0) < \mathrm{ctg}\,\alpha < \Phi'(1)$, there is a fully "illuminated" region near the nose of the body (i.e. a value of a exists such that the part of the lateral surface cut off by the planes $x_1 = 0$ and $x_1 = a$ is fully "illuminated"), which changes into a "partially illuminated" region (i.e. there exist a_1 and a_2 such that for $a_1 < a_* < a_2$ the circle representing the intersection of the plane $x_1 = a_*$ with the surface of the body is partially located in the aerodynamic shadow). For $\mathrm{ctg}\,\alpha < \Phi'(0)$, there is only "partial illumination" of the lateral surface. In the last case, which is considered below in detail, the region σ is described by

$$\theta^{(1)}(\omega, \rho) \leqslant \theta \leqslant \theta^{(2)}(\omega, \rho), \quad 0 \leqslant \rho \leqslant 1$$

and the functions $\theta^{(\nu)}(\omega, \rho)$ defining the boundary of the region satisfy the equation

$$t\,[\theta^{(\nu)}(\omega, \rho)] = 0, \quad \nu = 1, 2 \tag{6}$$

Rotational derivatives are the coefficients of the Taylor (Maclaurin) series expansion of the corresponding force or moment characteristic treated as a function of the angular velocity components. Thus, a knowledge of the rotational derivatives enables us to analyse the effect of rotation on the aerodynamic characteristics of the body in complex motion.

The calculation of rotational derivatives is a difficult problem primarily because the region $\sigma(S)$, as well as the integrand in (2), (3), depend on the angular velocity component ω_i. Therefore, the analysis of particular gas flow models usually requires simplifying assumptions [1–3]. However, as we show below, exact expressions for the rotational derivatives can be obtained even in the general case of model (1).

The formulas previously obtained in [7] for the first-order rotational derivatives can be easily transformed to our case. This enables us to proceed directly to the calculation of the second-order rotational derivatives.

We will use the following formula for differentiation with respect to a parameter of the integral of some function ψ:

$$\begin{aligned} &\frac{\partial^2}{\partial\omega_j\partial\omega_k}\iint_\sigma \psi(\omega,\rho,\theta)\,d\theta\,d\rho = \iint_\sigma \frac{\partial^2\psi}{\partial\omega_j\partial\omega_k}\,d\theta\,d\rho + \\ &+\int_0^1 \sum_{\nu=1}^{2}(-1)^\nu\Bigg[\frac{\partial\psi(\omega,\rho,\theta^{(\nu)})}{\partial\omega_j}\frac{\partial\theta^{(\nu)}}{\partial\omega_k} + \frac{\partial\psi(\omega,\rho,\theta^{(\nu)})}{\partial\omega_k}\frac{\partial\theta^{(\nu)}}{\partial w_j} + \\ &+ 2\,\frac{\partial\psi(\omega,\rho,\theta)}{\partial\theta}\bigg|_{\theta=\theta^{(\nu)}}\frac{\partial\theta^{(\nu)}}{\partial\omega_j}\frac{\partial\theta^{(\nu)}}{\partial\omega_k} + \psi(\omega,\rho,\theta^{(\nu)})\frac{\partial^2\theta^{(\nu)}}{\partial\omega_j\partial\omega_k}\Bigg]d\rho \end{aligned} \tag{7}$$

On the right-hand side of (7), the second integral is associated with the variation of the boundaries of the region σ. We can similarly represent the rotational derivatives in the form

$$\begin{aligned} c_i^{jk} &= \partial^2 c_{x_i}/\partial\omega_j\partial\omega_k = C_i^{jk} + \Delta C_i^{jk} \\ m_i^{jk} &= \partial^2 m_{x_i}/\partial\omega_j\partial\omega_k = M_i^{jk} + \Delta M_i^{jk} \end{aligned}$$

where Δ is a correction due to the variation of the boundary of σ.

Henceforth, the rotational derivatives are considered at $\omega = 0$ and the corresponding parameters are subscripted 0.

Using the symmetry properties, the expressions for the rotational derivatives of the force components can be written in the form

$$C_{0i}^{jk} = \iint_{\sigma_0} A_{0i}^{jk} d\rho\, d\theta = 2 \iint_{\Sigma_0} A_{0**i}^{jk}\, d\rho\, d\theta \tag{8}$$

$$A_{0i}^{jk} = (\partial^2 A_i/\partial\omega_j \partial\omega_k)_{\omega=0} = A_{0*i}^{jk} + A_{0**i}^{jk} \tag{9}$$

where Σ_0 is the part of the region σ_0 with $x_3 \geqslant 0$; one and two asterisks denote the components of the function that are, respectively, odd and even in θ.

The formulas for A_{0*i}^{jk}, A_{0**i}^{jk} can be obtained by direct differentiation of (4) using the relationships

$$(\partial\Omega_\nu/\partial\omega_j)_{\omega=0} = \mu_4 \Omega_{0\nu}' \,(-\sin\theta\delta_2{}^j + \cos\theta\delta_3{}^j)$$

$$(\partial^2\Omega_\nu/\partial\omega_j\partial\omega_k)_{\omega=0} = \mu_4{}^2\Omega_\nu''(\sin^2\theta\delta_2{}^{jk} + \cos^2\theta\delta_3{}^{jk} - \sin\theta\cos\theta\delta_{2,3}^{jk}),\quad \mu_4 = \mu_2/\mu_1$$

$$\Omega_{0\nu}' = d\Omega_\nu\,[t\,(\omega, \rho, \theta)]/dt\,|_{\omega=0},\quad \Omega_{0\nu}'' = d^2\Omega_\nu\,[t\,(\omega, \rho, \theta)]/dt^2\,|_{\omega=0}$$

and can be represented in the form

$$A_{0*1}^{jk} = (\mu_4{}^2\Omega_{01}'' + \mu_2\mu_4 v_\infty \cos\alpha\Omega_{02}'' + 2\mu_2\rho\Omega_{02}')\rho\sin\theta\cos\theta\delta_{2,3}^{jk}$$

$$A_{0*2}^{jk} = (-\mu_4{}^2\Phi'\cos^2\theta\Omega_{01}'' - \mu_2\mu_4 v_\infty \sin\alpha\cos\theta\Omega_{02}'' -$$
$$- \mu_2\Phi\Omega_{02}')\rho\sin\theta\delta_{2,3}^{jk} - \mu_2\rho^2\sin\theta\cos\theta\Omega_{02}'\delta_{1,3}^{jk}$$

$$A_{0*3}^{jk} = (\mu_4{}^2\Phi'\sin^2\theta\Omega_{01}'' + 2\mu_2\Phi\Omega_{02}')\rho\sin\theta\delta_2{}^{jk} +$$
$$+ \mu_4{}^2\Phi'\Omega_{01}''\sin\theta\cos^2\theta\rho\delta_3{}^{jk} - \mu_2\rho^2\Omega_0'\sin\theta\cos\theta\delta_{1,2}^{jk}$$

$$A_{0**1}^{jk} = -\rho\,(\mu_4{}^2\Omega_{01}'' + \mu_2\mu_4 v_\infty\cos\alpha\Omega_{02}'' + 2\mu_2\rho\Omega_{02}')(\delta_2{}^{jk}\sin^2\theta + \delta_3{}^{jk}\cos^2\theta)$$

$$A_{0**2}^{jk} = (\mu_4{}^2\cos\theta\Omega_{01}'' + \mu_2\mu_4 v_\infty\sin\alpha\Omega_{02}'')\rho\sin^2\theta\delta_2{}^{jk} +$$
$$+ (\mu_2\mu_4 v_\infty\sin\alpha\cos\theta\Omega_{02}'' + 2\mu_2\Phi\Omega_{02}' + \mu_4\Phi'\cos\theta\Omega_{01}'')\,\rho\cos\theta\delta_3{}^{jk} +$$
$$+ \mu_2\rho^2\sin^2\theta\Omega_{02}'\delta_{1,2}^{jk}$$

$$A_{0**3}^{jk} = (\mu_4{}^2\Phi'\sin^2\theta\Omega_{01}'' - \mu_2\Phi\Omega_{02}')\,\rho\cos\theta\delta_{2,3}^{jk} + \mu_2\rho^2\cos^2\theta\Omega_{02}'\delta_{1,3}^{jk}$$

where $\delta_\nu{}^{jk} = 1$ if $j = k = \nu$ and $\delta_\nu{}^{jk} = 0$ otherwise; $\delta_{\nu,\mu}{}^{jk} = 1$, if $j = \nu$ and $k = \mu$ or $\mu = j$ and $k = \nu$, and $\delta_{\nu,\mu}{}^{jk} = 0$ otherwise.

Using relationship (4) and a representation similar to (9), we write the formulas for the rotational derivatives of the moment characteristics

$$M_{0i}{}^{jk} = 2\iint_{\Sigma_0} B_{0**i}^{jk}\, d\rho\, d\theta \tag{10}$$

$$B_{0**i}^{jk} = \rho\,(A_{0**3}^{jk}\cos\theta - A_{0*2}^{jk}\sin\theta)\,\delta_i{}^1 +$$
$$+ (\rho A_{0*1}^{jk}\sin\theta - \Phi A_{0**3}^{jk})\delta_i{}^2 + (\Phi A_{0**2}^{jk} - \rho\cos\theta A_{0**1}^{jk})\delta_i{}^3$$

The corrections $\Delta C_{0i}{}^{jk}$, $\Delta M_{0i}{}^{jk}$ are given by the second integrals in (7) evaluated respectively for $\chi = A_i$ and $\chi = B_i$ for $\omega = 0$. The required formulas for $\theta^{(\nu)}$ and their derivatives can be obtained from (6). The resulting expressions are

$$\theta_0^{(2)} = \theta_0(\rho),\quad \theta_0^{(1)} = -\theta_0(\rho),\quad \sin\theta_0 = \frac{\mu_5}{\Phi'},\quad \cos\theta_0 = -\frac{\operatorname{ctg}\alpha}{\Phi'}$$

$$\left.\frac{\partial\theta^{(\nu)}(\omega,\rho)}{\partial\omega_j}\right|_{\omega=0} = \theta_{0**}^{j} + (-1)^\nu\theta_{0*}^{j},\qquad \left.\frac{\partial^2\theta^{(\nu)}}{\partial\omega_j\partial\omega_k}\right|_{\omega=0} = \theta_{0**}^{jk} + (-1)^\nu\theta_{0*}^{jk}$$

$$\theta_{0**}^{j} = -\frac{\mu_2}{\mu_3}\delta_2{}^j,\quad \theta_{0*}^{j} = -\frac{\mu_2}{\mu_3\mu_5}\operatorname{ctg}\alpha\delta_3{}^j$$

$$\theta_{0*}^{jk} = -\frac{\mu_2{}^2\operatorname{ctg}\alpha}{\mu_3{}^2\mu_5{}^3}\,[\mu_5{}^2\delta_2^{jk} - \mu_3\,(2\Phi'^2 - \operatorname{ctg}^2\alpha)\,\delta_3^{jk}]$$

$$\theta_{0**}^{jk} = \frac{\mu_2{}^2}{\mu_3{}^2}\left(\delta_{2,3}^{jk} - \frac{\mu_3}{\mu_2\mu_5{}^2}\operatorname{ctg}^2\alpha\delta_{1,3}^{jk}\right),\quad \mu_5 = \sqrt{\Phi'^2 - \operatorname{ctg}^2\alpha}$$

Allowing for the symmetry properties, we obtain

$$\Delta C_{0i}^{jk} = 2\int_0^1 [\theta_{0**}^k A_{0*i}^j + \theta_{0**}^j A_{0*i}^k + \theta_{0*}^k A_{0**i}^j + \theta_{0*}^j A_{0**i}^k + 2(\theta_{0*}^k\theta_{0**}^j + \theta_{0*}^j\theta_{0**}^k) \times$$
$$\times A_{0**i}^{\theta} + 2(\theta_{0**}^k\theta_{0**}^j + \theta_{0*}^k\theta_{0*}^j) A_{0*i}^{\theta} + \theta_{0**}^{jk} A_{0*i} + \theta_{0*}^{jk} A_{0**i}]\, d\rho \tag{11}$$

The expressions for $\Delta M_{0i}{}^{jk}$ are obtained by replacing the symbol A by the symbol B in (11), and

$$\begin{gathered}
A_{0i}^{\theta} = (\partial A_i/\partial\theta)_{\omega=0} = A_{0*i}^{\theta} + A_{0**i}^{\theta}, \quad B_{0i}^{\theta} = B_{0*i}^{\theta} + B_{0**i}^{\theta} \\
A_{0*1}^{\theta} = \mu_7\rho\,(\Omega_{01}' + \mu_1 v_\infty \cos\alpha\Omega_{02}')\sin\theta, \quad A_{0**1}^{\theta} = 0 \\
A_{0*2}^{\theta} = -\rho\,(\Phi'\Omega_{01}' + \mu_7\Phi'\cos\theta\Omega_{01}' + \mu_3 v_\infty \sin\alpha\Omega_{02}') \times \\
\times \sin\theta, \; A_{0**2}^{\theta} = 0 \\
A_{0*3}^{\theta} = 0, \; A_{0**3}^{\theta} = \rho\Phi'(\cos\theta\Omega_{01} - \mu_7 \sin^2\theta\Omega_{01}') \\
B_{0*1}^{\theta} = 0, \; B_{0**1}^{\theta} = \rho\,[(A_{0**3}^{\theta} + A_{0**2})\cos\theta + \\
+ (A_{0*2}^{\theta} - A_{0*3})\sin\theta]
\end{gathered} \tag{12}$$

$$\begin{gathered}
B_{0*2}^{\theta} = 0, \; B_{0**2}^{\theta} = \rho\,(A_{0**1}\cos\theta + A_{0*1}^{\theta}\sin\theta) - \Phi A_{0**3}^{\theta} \\
B_{0*3}^{\theta} = \Phi A_{0*2}^{\theta} - \rho\sin\theta A_{0**1} - \rho\cos\theta A_{0**1}^{\theta}, \quad B_{0**3}^{\theta} = 0 \\
A_{0*1} = 0, \; A_{0**1} = -\rho\,(\Omega_{01} + \mu_1 v_\infty \cos\alpha\Omega_{02}) \\
A_{0*2} = 0, \; A_{0**2} = \rho\,(\Phi'\cos\theta\Omega_{01} + \mu_1 v_\infty \sin\alpha\Omega_{02}) \\
A_{0*3} = \rho\Phi'\sin\theta\Omega_{01}, \; A_{0**3} = 0 \\
B_{0*i} = \rho\,(\cos\theta A_{0*3} - \sin\theta A_{0**2})\,\delta_i{}^1 + (\rho\sin\theta A_{0**1} - \\
- \Phi A_{0*3})\,\delta_i{}^2, \; B_{0**i} = (\Phi A_{0**2} - \rho\cos\theta A_{0**1})\,\delta_i{}^3
\end{gathered}$$

Expressions (8) and (10)–(12) give a "folded" representation for the rotational derivatives; if we "unfold" the corresponding formulas, we find that many rotational derivatives vanish and their matrices have the form

$$C_{01}^{jk}: \begin{Vmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{Vmatrix}, \quad C_{02}^{jk}: \begin{Vmatrix} 0 & + & 0 \\ + & * & 0 \\ 0 & 0 & * \end{Vmatrix}$$

$$C_{03}^{jk}, \quad m_{01}^{jk}, \quad m_{02}^{jk}: \begin{Vmatrix} 0 & 0 & * \\ 0 & 0 & * \\ * & * & 0 \end{Vmatrix}, \quad m_{03}^{jk}: \begin{Vmatrix} 0 & + & 0 \\ + & * & 0 \\ 0 & 0 & + \end{Vmatrix}$$

where the symbols "plus" and "star" are the non-zero values of the rotational derivatives. The star corresponds to a non-zero (in general) correction associated with the variation of the "illuminated" part. Analysis has shown that these corrections do not produce new non-zero rotational derivatives. Comparison with the results of [3], which used the free-molecular flow approximation, shows that the lists of zero derivatives match.

We should emphasize an important aspect associated with the transition to the general class of local interaction models: we have shown that the rotational derivatives preserve their zero value when the flow conditions are varied over a wide range in the framework of model (1).

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QUASI-MONOCHROMATIC WEAKLY NON-LINEAR WAVES IN A LOW-DISPERSION BUBBLE MEDIUM†

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The propagation of quasi-monochromatic wave packets in a rarefied polydispersed mixture of a weakly compressible liquid with a finite number of fractions of differently sized gas bubbles is considered. Two equations for the modulation waves are derived by the multi-scale method in the cubic approximation in the wave amplitude: the non-linear Schrödinger equation ignoring dissipation effects and the Landau–Ginzburg equation for low dissipation due to the viscosity of the liquid and heat losses associated with bubble vibration. The coefficients of the non-linear Schrödinger equation are investigated to analyse the non-linear (modulational) stability of waves in a monodispersed non-dissipative bubble medium.

A LINEAR dispersion relationship has been previously obtained for acoustic waves in a polydispersed bubble medium without dissipation [1] and for waves in a dissipative medium [2]. The general scheme for deriving the amplitude equations by the asymptotic multiscale method has been described in several monographs (see, e.g. [3]). Modulation equations have been obtained [4] for waves in a monodispersed bubble chamber by Whitham's averaged Lagrangian method [5].

1. THE EQUATIONS OF MOTION IN A NON-DISSIPATIVE MEDIUM

The plane one-dimensional motion of an ideal weakly compressible liquid with a low volume content of spherical gas bubbles ($\alpha_g \ll 1$) under conditions when thermal dissipation and capillary effects can be ignored is described by Iordanskii's equations [1, 6, 7]

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